CHRISTIAN SOCIAL SERVICES COMMISSION (CSSC)

NORTHERN ZONE JOINT EXAMINATION SYNDICATE (NZJES).

FORM FOUR PRE-NATIONAL EXAMINATIONS 2024

042 ADDITIONAL MATHEMATICS

MARKING SCHEME

1 (a) (i) Given $M^2 \propto \frac{1}{\sqrt[4]{N}}$

Introducing proportionality constant

$$M^2 = \frac{K}{\sqrt[4]{N}}$$

when $M=12$, $N=16$

$$(12)^2 = \frac{K}{\sqrt[4]{16}}.$$

$$144 = \frac{K}{2} \Rightarrow K = 288$$

The equation connecting M and N (01 mrk)

$$M^2 = \frac{288}{\sqrt[4]{N}}$$

$$\sqrt[4]{N} = \frac{288}{M^2}$$

$$N = \sqrt[4]{\frac{288}{M^2}}$$

(ii) The value of M when $N=81$

From $M^2 = \frac{288}{\sqrt[4]{N}}$ (01 mrk)

1 (a) (ii) $M^2 = \frac{288}{\sqrt[4]{81}}$

$M^2 = \frac{288}{3}$

$M^2 = 96$ (01mrk)

$M = 4\sqrt{6} = 9.797959$

$\therefore$ The value of M is $4\sqrt{6}$.

(b) Let n be the amount of fuel
d be the distance travelled
v be the speed of the train

$n \propto dv^2$

$n = kdv^2$ (01mrk)

when $n = 20L$, $d = 160\,km$, $v = 80\,km/hr$

$n = kdv^2$

$\Rightarrow k = \left(\frac{dv^2}{n}\right)^{-1}$

$k = \left(\frac{160km \times (80km/hr)^2}{20L}\right)^{-1}$

$k = \frac{1}{51200}$ (01 mark)

now,

$n = \frac{dv^2}{51200}$

when $d = 320\,km$, $v = 40\,km/hr$ ($\frac{01}{2}$ mark)

$n = \frac{320km \times (40\,km/hr)^2}{51200}$

$n = 10$ litres. ($\frac{01}{2}$ mark)

$\therefore$ The amount of fuel used is 10 Litres.

2 (a) let m denote the mass value

Given 9kg, 10kg, 12kg, a, and b

Mean masses ($\bar{m}$) = 8 kg

no of students = 5

From

$$\bar{m} = \frac{\Sigma m}{\Sigma F}$$

(01 marks)

$$8 = \frac{31 + a + b}{5}$$

$a + b = 9$ --- (i)

but the product of a and b is 20

$ab = 20$ -- (ii)

From,

$$ab = 20$$

$$a = \frac{20}{b}$$

(01 marks)

$$\frac{20}{b} + b = 9$$

$$20 + b^2 = 9b$$

$$b^2 - 9b + 20 = 0$$

on solving

$b = 5$ or $b = 4$

(01 mark)

when $b = 5$, $a = 4$

when $b = 4$, $a = 5$

∴ The masses of the other two students is 4kg and 5kg respectively.

2 (b)

Age	frequency(F)	Fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$F(x-\bar{x})^2$
2	6	12	-2.9	8.41	50.46
3	8	24	-1.9	3.61	28.88
4	17	68	-0.9	0.81	13.77
5	21	105	0.1	0.01	0.21
6	15	90	1.1	1.21	18.15
7	11	77	2.1	4.41	48.51
8	2	16	3.1	9.61	19.22

(01 marks)

$\Sigma F = 80$ $\Sigma fx = 392$ $\Sigma F(x-\bar{x})^2 = 179.2$

(i) Mean

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{392}{80}$$

$$\bar{x} = 4.9$$ (01 mark)

(ii) variance

$$Var(x) = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$$

$$= \frac{179.2}{80}$$

$$= 2.24$$

$\therefore$ variance $= 2.24$ (01 marks)

3 (a) Given the equation

$3x^2 - 10xy + 7y^2 = 0$

factorizing

$3x(x-y) - 7y(x-y) = 0$

$(3x-7y)(x-y) = 0$ (01 mrk)

$3x - 7y = 0$ and $x - y = 0$

$3x = 7y$ and $x = y$

The epuations of the lines are

$y = \frac{1}{3}x$ and $y = x$

The slopes of the lines are $M_1 = \frac{1}{3}$ and $M_2 = 1$

from,

$$\tan\theta = \frac{M_1 - M_2}{1 + M_1 M_2}$$

$$\tan\theta = \frac{1 - \frac{1}{3}}{1 + \left(\frac{1}{3}\right)(1)}$$ (01 mrk)

$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$ (01 mark)

$$\theta = 26.57^\circ$$

$\therefore$ The angle between the lines is 26.57°

3 (b) Given the equation of the boundary of the garden

$4x^2+4y^2-40x-48y+48=0$

$x^2+y^2-10x-12y+12=0$

$x^2-10x+y^2-12y=-12$ (01 mrk)

Factorizing

$(x-5)^2+(y-6)^2=-12+25+36$

$(x-5)^2+(y-6)^2=49$ (01 mrk)

$(x-5)^2+(y-6)^2=7^2$

Comparing with the standard eqn of circle

$(x-a)^2+(y-b)^2=r^2$

$r^2=7^2 \Rightarrow r=7$ (01 mrk)

radius $=7$ units

$\therefore$ The watering pump should be placed at 7 units from the boundary of the garden.

4 (a) Recall, $\overline{PS}=\overline{PN}$

$\sqrt{(x-q)^2+y^2}=\sqrt{(x+q)^2}$

$(x-q)^2+y^2=(x+q)^2$ (01 mrk)

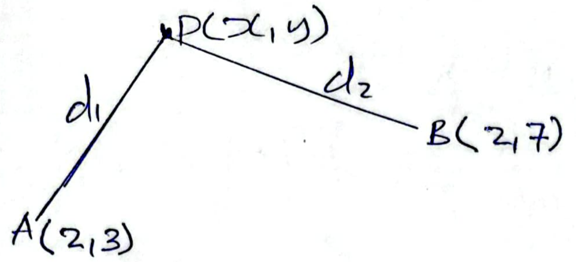
$x^2-2qx+q^2+y^2=x^2+2q+q^2$ (01 mrk)

$y^2=2qx+2qx$ (01 mrk)

$\therefore y^2=4qx$

Shown!

4 (b) Let the coordinate of the point be $P(x,y)$



($\frac{01}{2}$ mrk)

From, $d_1 = d_2$

$$\sqrt{(x-2)^2+(y-3)^2} = \sqrt{(x-2)^2+(y-7)^2}$$ ($\frac{01}{2}$ mrk)

squaring both sides

$$(x-2)^2+(y-3)^2 = (x-2)^2+(y-7)^2$$ (01 mrk)

on expanding and simplifying

$$-6y+9 = -14y+49$$

$$8y = 40$$

$$y = 5$$ (01 mrk)

$\therefore$ The locus of a goat is $y=5$

5 (a) Given

$$\frac{1}{y}+y = 2\sqrt{5}$$

Square both sides (01 mrk)

$$\left(\frac{1}{y}+y\right)^2 = (2\sqrt{5})^2$$

$$\frac{1}{y^2}+y^2+2 = 20$$ (01 mrk)

$$\frac{1}{y^2}+y^2 = 18$$

$\therefore$ The value of $\frac{1}{y^2}+y^2$ is 18

5 (b) From,

$x : y = 5 : 1$

$\frac{x}{y} = \frac{5}{1} \Rightarrow x = 5y$

From

$\frac{x+y}{3x-4y}$ ——— $\left(\frac{01}{2}\right)$

$= \frac{5y+y}{3(5y)-4y}$ ——— $\left(\frac{01}{2}\right)$

$= \frac{6y}{11y}$

$= \frac{6}{11}$

$\therefore$ The value of $\frac{x+y}{3x-4y}$ is $\frac{6}{11}$ ——— (01)

(c) Given

$\log_x y = 2$ and $xy = 8$

changing the logarithmic eqn to exponential Eqn

$x^2 = y$ --- ①

$xy = 8$ - ⑪ ——— (01 mrk)

Substitute eqn ① into eqn ⑪

$x(x^2) = 8$

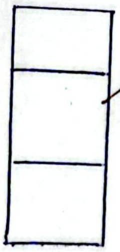
$x^3 = 2^3$ ——— $\left(\frac{01}{2}\text{mrk}\right)$

$x = 2$

but $y = x^2 \Rightarrow y = 2^2 = 4$

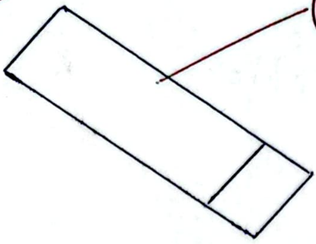
$\therefore$ $x = 2$ and $y = 4$ ——— $\left(\frac{01}{2}\text{mrk}\right)$

6 (a)



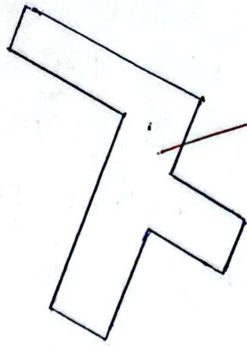
(01)

Front



(01)

Plan



(01)

Side

(b) From the given information

Exterior angle $= P$

Interior angle $= 3P$

$P + 3P = 180^\circ$

$4P = 180^\circ$

$P = 45^\circ$ — 01

(i) $\dfrac{6P - 16^\circ}{2} = \dfrac{6(45^\circ) - 16^\circ}{2}$

$= 127^\circ$

$\therefore \dfrac{6P - 16^\circ}{2} = 127^\circ$ — $\frac{01}{2}$

(ii) Interior angle $= 3P = 3(45^\circ)$

$= 135^\circ$ — $\frac{01}{2}$

$\therefore$ The size of interior angle is $\underline{135^\circ}$

(iii) From, $n = \dfrac{360^\circ}{P} = \dfrac{360^\circ}{45^\circ}$, $n = 8$

Sum of interior angles $= (n-2)\,180^\circ$

$= (8-2)\,180^\circ$

$\therefore$ Sum of interior angles $= 1080^\circ$ — 01

09

7 (a) Given,

$$y = \frac{1+\sin\theta}{\cos\theta}$$

rationalize the numerator of R.H.S

$$y = \frac{1^2 - \sin^2\theta}{\cos\theta - \cos\theta\sin\theta}$$

Reciprocate both sides (01)

$$\frac{1}{y} = \frac{\cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta}$$

but $\sin^2\theta = 1-\cos^2\theta$

$$\frac{1}{y} = \frac{\cos\theta - \cos\theta\sin\theta}{1-(1-\cos^2\theta)}$$ (01)

$$\frac{1}{y} = \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$ (01)

$$\frac{1}{y} = \frac{1-\sin\theta}{\cos\theta}$$

Shown!!

(b) Given

$\tan^3 A + 2\tan^2 B + 3 = 0$

Consider $\tan A = \frac{\sin A}{\cos A}$, $\tan B = \frac{\sin B}{\cos B}$

$$\frac{\sin^2 A}{\cos^2 A} + \frac{2\sin^2 B}{\cos^2 B} + 3 = 0$$ (01)

$$\frac{\sin^2 A\cos^2 B + 2\cos^2 A\sin^2 B + 3\cos^2 A\cos^2 B}{\cos^2 A\cos^2 B} = 0$$ (01)

$$\sin^2 A\cos^2 B + 2\cos^2 A\sin^2 B + 3\cos^2 A\cos^2 B = 0$$

7 (b) but $\sin^2 A = 1 - \cos^2 A, \quad \sin^2 B = 1 - \cos^2 B$

$$\cos^2 B(1-\cos^2 A) + 2\cos^2 A(1-\cos^2 B) + 3\cos^2 A\cos^2 B = 0$$

$$\cos^2 B + 2\cos^2 A - 3\cos^2 A\cos^2 B + 3\cos^2 A\cos^2 B = 0$$

$$\cos^2 B + 2\cos^2 A = 0 \quad \text{———} \quad (01)$$

Proved!

8 (a) From the formula of sum of n terms of A.P

$$S_n = \frac{n}{2}(2A_1 + (n-1)d)$$

For odd numbers $A_1 = 1$ and $d = 2$

$$S_n = \frac{n}{2}(2 + (n-1)2) \quad \text{———} \quad (01)$$

$$S_n = \frac{n}{2}(2 + 2n - 2)$$

$$S_n = n^2 \quad \text{———} \quad (01)$$

$\therefore$ The rule governing the pattern of the sum of all odd numbers is n^2. ——— (01)

(b) The number is divisible by 5 if its last digit is 0 or 5
The last digit of 33775 is 5, hence it is divible by 5
The number is divisible by 6 if it is divisible by 2 and 3 separately ——— (01)
The number 33775 is neither divisible by 2 nor divisible by 3, hence it is not divisible by 6. ——— (02)

9 (a) By using truth table

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T

(02)

Since the truth values in the column of $\neg(p \wedge q) \vee (p \vee q)$ in the truth table contain only the truth values T, then it is tautology.

Therefore, $(p \wedge q) \rightarrow (p \vee q)$ is tautology (01)

(b). Let p – I read my exercise book

q – I will pass my examination

statement (argument) in symbolic form

$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ 01

using the laws of algebra of proposition to simplify

$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ --- Given

$[(\neg p \vee q) \wedge \neg q] \rightarrow \neg p$ --- Implication law

$[(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \rightarrow \neg p$ --- Distributive law

$((\neg p \wedge \neg q) \vee F) \rightarrow \neg p$ ---- Complement law

$(\neg p \wedge \neg q) \rightarrow \neg p$ ---- identity law 02

$(p \vee q) \vee \neg p$ ---- Implication & De Morgan's

$(p \vee \neg p) \vee q$ ---- Associative law

$T \vee q$ ---- complement law

T ---- identity law

∴ The argument is valid, since it is tautology.

10 (a) (i) $(A \cup B)' \cap A$ — Given

$(A' \cap B') \cap A$ -- De Morgan's law — 01

$\phi \cap B'$ — — Associative & complement law

ϕ — — — Identity law — 01

$\therefore (A \cup B)' \cap A \equiv \phi$

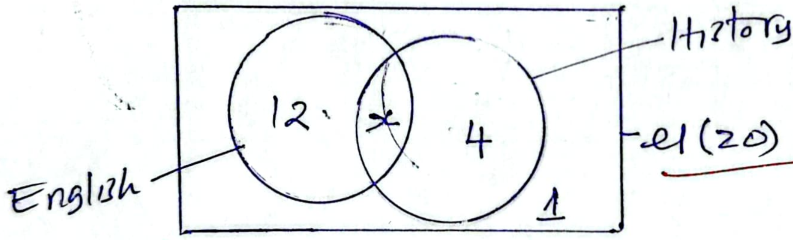
(ii) $(A - B) \cup A$ — Given

$(A \cap B') \cup A$ --- Set difference

A — — — — Absorption

$\therefore (A - B) \cup A \equiv A.$ — 01

(b) By using Venn diagram



— 01

Then, $12 + x + 4 + 1 = 20$

$x = 3$ — 01

History $= 3 + 4 = 7$

$\therefore$ The number of pupils studying history is 7 — 01

11 (a) Given

$(2P+2)x^2+Px+P = 4(Px+2)$

$(2P+2)x^2-3Px+P-8 = 0$

$a = 2P+2, \quad b = -3P \quad c = P-8$

Sum of roots $\alpha+\beta = -\frac{b}{a} = \frac{3P}{2P+2}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{P-8}{2P+2}$ — 01

Since $\alpha+\beta = \alpha\beta$

$\frac{3P}{2P+2} = \frac{P-8}{2P+2}$ — 01

$3P = P-8$

$2P = -8$

$P = -4$

$\therefore$ The value of P is -4. — 01

(b) Given

$g(x) = \frac{x+3}{x-2}$

Vertical asymptote (V.A)

$x-2=0$ — 01

$x = 2$

Horizontal asymptote (H.A)

$y = \lim_{x\to\infty} \frac{x+3}{x-2}$ — 01

$y = \lim_{x\to\infty} \frac{1+3/x}{1-2/x} = 1, \quad y=1$

11 (b) y-intercept ($x=0$)

$$y = \frac{0+3}{0-2} = -3/2$$

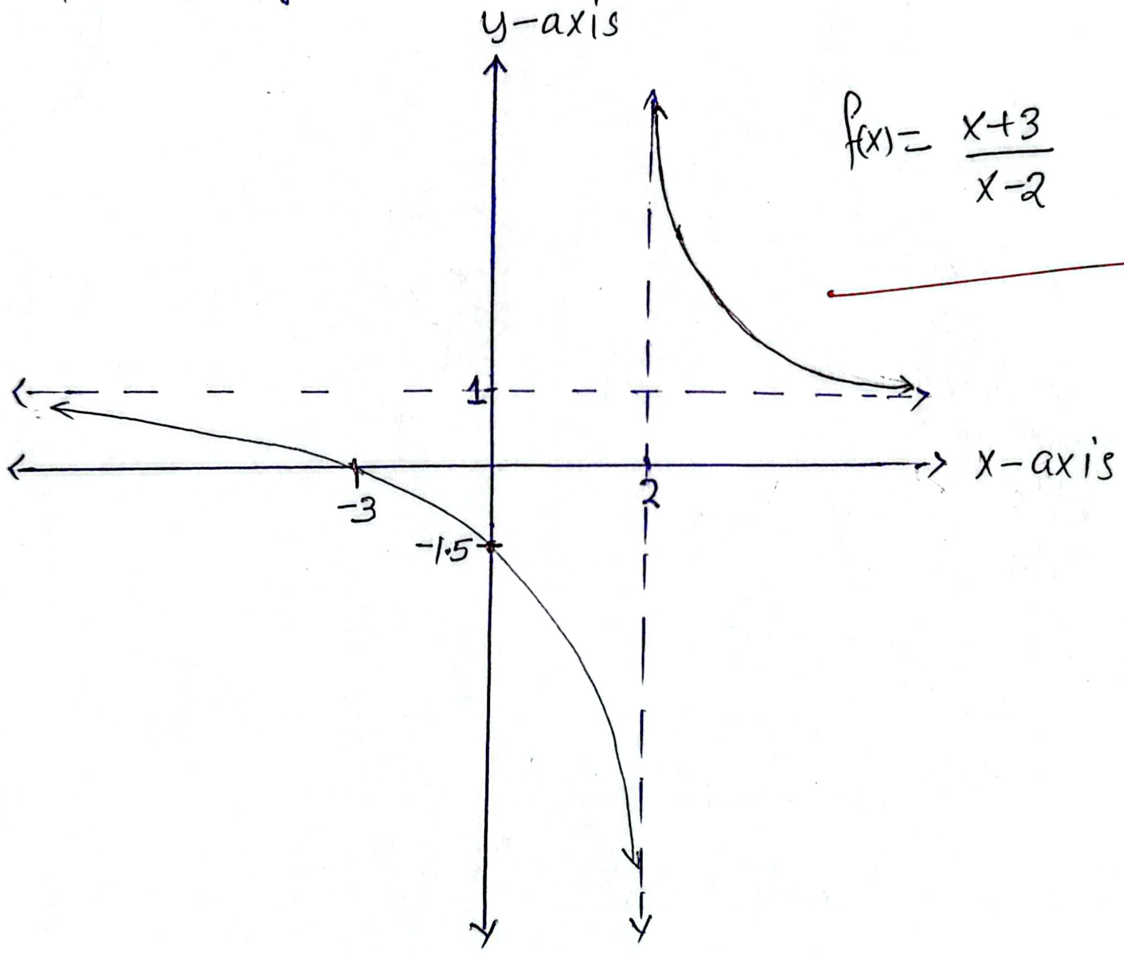
x-intercept ($y=0$)

$$0 = \frac{x+3}{x-2}$$ 01

$$x+3=0$$

$$x=-3$$

The graph



02

(c) Domain $= \{x : x \in \mathbb{R}, x \neq 2\}$

Range $= \{y : y \in \mathbb{R}, y \neq 1\}$ 02

12 (a) Velocity $= \frac{ds}{dt}$

$s = 9t^2 - 2t^3$

$\frac{ds}{dt} = 18t - 6t^2$ ———— $0\frac{1}{2}$

at $t = 3$

$\frac{ds}{dt} = 18(3) - 6(3)^2$

$= 54 - 54$

$\frac{ds}{dt} = 0\, m/s$ ———— $0\frac{1}{2}$

$\therefore$ Velocity at $t=3$ is $0\, m/s$

Acceleration $= \frac{dv}{dt} = \frac{d^2s}{dt^2}$ ———— 01

$\frac{d^2s}{dt^2} = 18 - 12t$

at $t = 3$

$\frac{d^2s}{dt^2} = 18 - 12 \times 3$ ———— 01

$= -18\, m/s^2$

$\therefore$ The acceleration $= -18\, m/s^2$.

(b) Given

$\int_0^2 x(a-x)^2\, dx = \frac{2}{3}$

$\int_0^2 x(a^2 - 2ax + x^2)\, dx = \frac{2}{3}$

12 (b) $\int_0^2 (a^2x - 2ax^2 + x^3)\,dx = \frac{2}{3}$ — 01/2

$\frac{a^2x^2}{2} - \frac{2ax^3}{3} + \frac{x^4}{4}\Big|_0^2 = \frac{2}{3}$ — 01

$\left(2a^2 - \frac{16a}{3} + 4\right) - 0 = \frac{2}{3}$

$6a^2 - 16a + 12 = 2$ — 01/2

$6a^2 - 16a + 10 = 0$

$3a^2 - 8a + 5 = 0$

on solving — 01

$a = 1$ or $\frac{5}{3}$

$\therefore$ The value of a is 1 or $\frac{5}{3}$. — 01

(c) From,

volume $= \int_a^b \pi y^2\,dx$ or $\int_a^b \pi x^2\,dy$

$V = \int_a^b \pi y\,dy$, $[x^2 = y]$ — 01

$V = \int_0^4 \pi y\,dy = \pi \int_0^4 y\,dy$ — 01

$V = \pi\left[\frac{y^2}{2}\right]_0^4 = \pi\left[\left(\frac{16}{2}\right) - \left(\frac{0}{2}\right)\right]$

$V = 8\pi$ cubic units — 01

$\therefore$ The volume $= 8\pi$ cubic units.

13 (a) Given

$$^nP_4 = 42(^nP_2)$$

$$\frac{n!}{(n-4)!} = \frac{42(n!)}{(n-2)!}$$

$$\frac{1}{(n-4)!} = \frac{42}{(n-2)(n-3)(n-4)!}$$ 01

$$\frac{1}{1} = \frac{42}{(n-2)(n-3)}$$

$$(n-2)(n-3) = 42$$ 01

$$n^2 - 5n + 6 - 42 = 0$$

$$n^2 - 5n - 36 = 0$$ 01

on solving

$$n = 9 \text{ or } -4$$

Since n can not be negative,

It follows that $n = 9$.

(b) $n = 14$, $r = 10$ 0½

but since the first 4 questions are compulsory

It follows that $n = 14 - 4 = 10$ 01

$r = 10 - 4 = 6$

No of choices $= {}^nC_r = {}^{10}C_6$

13 (b) no of choices = $\frac{10!}{(10-6)!\,6!}$ 01/2

$= \frac{10!}{4!\,6!}$

$= 210$ 01

$\therefore$ There are 210 choices.

(c) Let M be the event that stock market will go down

E be economy will deteriorate

$P(M/E) = 0.8$

$P(E) = 0.5$

$P(M \cap E) = ?$

Recall the conditional probability

$P(M \cap E) = P(M/E) \times P(E)$ 02

$= 0.8 \times 0.5$ 01

$= 0.4$

$\therefore$ The probability that the economy will deteriorate and the stock market will go down is 0.4 06

14 (a) Let matrix $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ — $0\frac{1}{2}$

$\det(A) = ad - bc$ — $0\frac{1}{2}$

Then, $k\det(A) = k(ad-bc)$

Also, from

$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$kA = k\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$kA = \begin{pmatrix} ka & kc \\ kb & kd \end{pmatrix}$ — 01

$\det(kA) = ka \cdot kd - kb \cdot kc$

$= k^2ad - k^2bc$ — 01

$= k^2(ad-bc)$

but $ad-bc = \det(A)$ — 01

$\det(kA) = k^2\det(A)$

$\therefore \det(kA) = k^2\det(A)$

Shown!

14 (b) From the formula of area of Parallelogram

$$\text{Area} = |\underset{\sim}{a} \times \underset{\sim}{b}| \text{ ——— } 01$$

let $\underset{\sim}{b} = -\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{a} = 6\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$

but $\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{J} & \underset{\sim}{K} \\ 6 & -1 & -1 \\ -1 & -4 & 2 \end{vmatrix}$ ——— 01

$$= (-2-4)\underset{\sim}{i} - (12-1)\underset{\sim}{J} + (-24-1)\underset{\sim}{K}$$

$\underset{\sim}{a} \times \underset{\sim}{b} = -6\underset{\sim}{i} - 11\underset{\sim}{j} - 25\underset{\sim}{K}$ ——— $\frac{01}{2}$

Thus,

$$|\underset{\sim}{a} \times \underset{\sim}{b}| = \sqrt{(-6)^2 + (-11)^2 + (-25)^2}$$

$$= \sqrt{782} \text{ ——— } \frac{01}{2}$$

$\therefore$ The area of Parallelogram is $\sqrt{782}$ square units

(c) Given, $3x + 4y + 6 = 0$

$$y = \frac{-3x}{4} - \frac{6}{4}$$

Select points on the line

A(2, -3) B(1, $-\frac{9}{4}$) ——— 01

Line $y = -x$, $\theta = 135^\circ$

14 (c) reflection matrix

$$M = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \qquad 0\tfrac{1}{2}$$

$$= \begin{pmatrix} \cos 270^\circ & \sin 270^\circ \\ \sin 270^\circ & -\cos 270^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$A' = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} \qquad 0\tfrac{1}{2}$$

$$A' = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$B' = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -9/4 \end{pmatrix}$$

$$B' = \left(\frac{9}{4}, -1 \right)$$

$$\text{slope} = \frac{-2+1}{3-9/4} = \frac{-4}{3} \qquad 01$$

Equation

$$\frac{-4}{3} = \frac{y+2}{x-3}$$

$$3y+6 = -4x+12$$

$$4x+3y-6 = 0 \qquad 01$$

$\therefore$ The image is $4x+3y-6=0$.